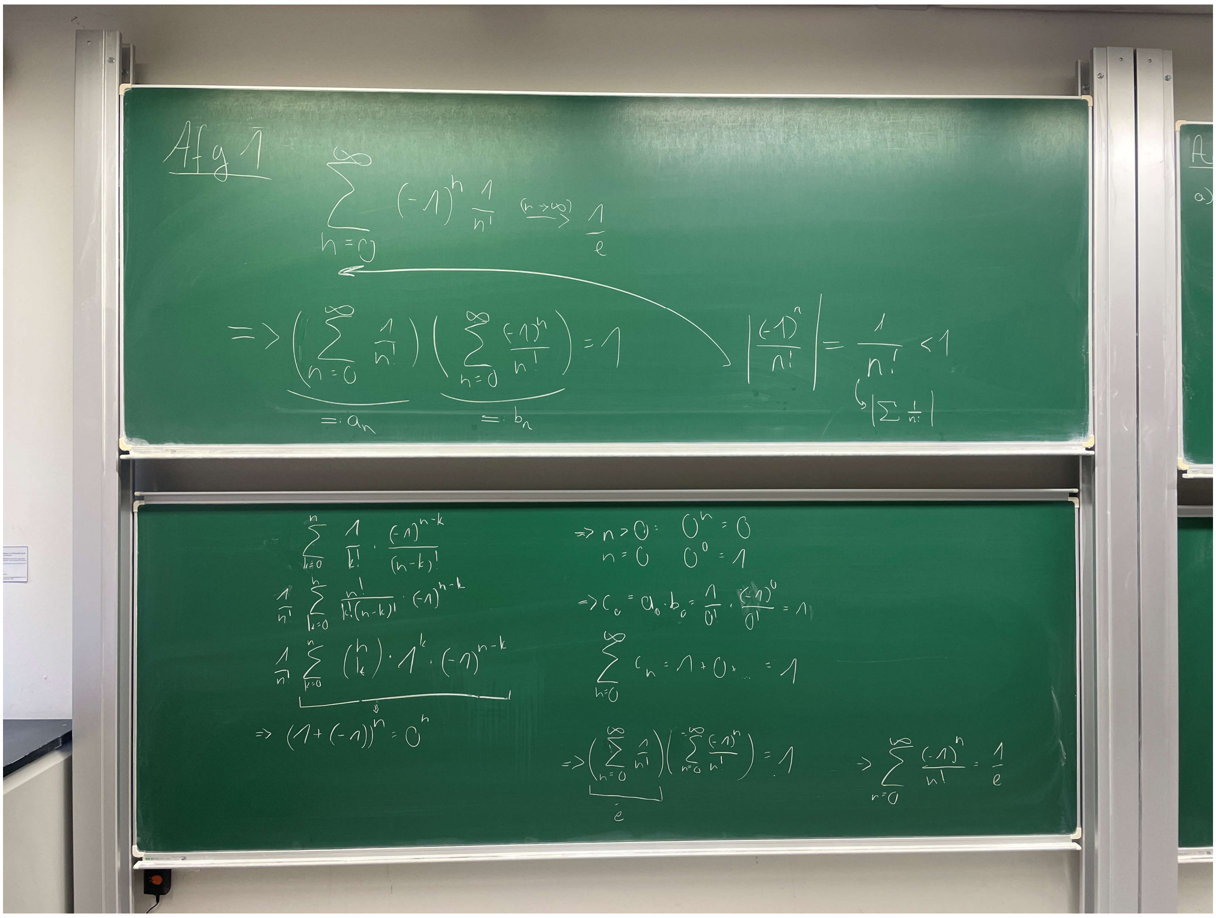
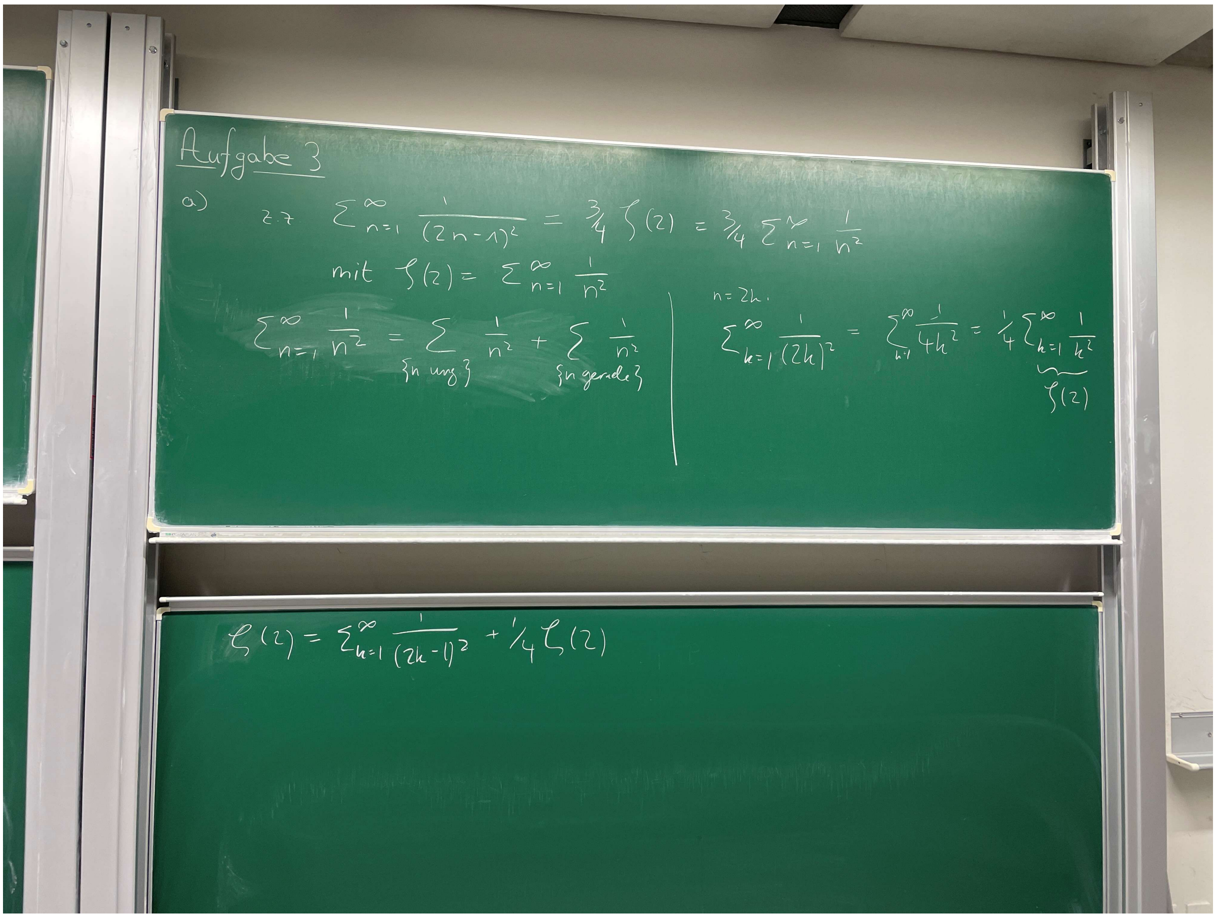

Afg 1
$\sum_{n=0}^{\infty} (-1)^n \frac{1}{n!} \xrightarrow{(n\to\infty)} \frac{1}{e}$
$\Rightarrow \underbrace{\left(\sum_{n=0}^{\infty} \frac{1}{n!}\right)}_{=: a_n} \underbrace{\left(\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\right)}_{=: b_n} = 1$
$\left|\frac{(-1)^n}{n!}\right| = \frac{1}{n!} < 1$
$\left|\sum \frac{1}{n!}\right|$
$\sum_{k=0}^{n} \frac{1}{k!} \cdot \frac{(-1)^{n-k}}{(n-k)!}$
$\frac{1}{n!} \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} \cdot (-1)^{n-k}$
$\frac{1}{n!} \underbrace{\sum_{k=0}^{n} \binom{n}{k} \cdot 1^k \cdot (-1)^{n-k}}$
$\Rightarrow (1+(-1))^n = 0^n$
$\Rightarrow n > 0: \quad 0^n = 0$
$n = 0 \quad 0^0 = 1$
$\Rightarrow c_0 = a_0 \cdot b_0 = \frac{1}{0!} \cdot \frac{(-1)^0}{0!} = 1$
$\sum_{n=0}^{\infty} c_n = 1 + 0 + \ldots = 1$
$\Rightarrow \underbrace{\left(\sum_{n=0}^{\infty} \frac{1}{n!}\right)}_{e} \left(\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\right) = 1$
$\Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$


Aufgabe 3
a) z.z. $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{3}{4} \zeta(2) = \frac{3}{4} \sum_{n=1}^{\infty} \frac{1}{n^2}$
mit $\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2}$
$\sum_{n=1}^{\infty} \frac{1}{n^2} = \sum_{\{n \text{ ung.}\}} \frac{1}{n^2} + \sum_{\{n \text{ gerade}\}} \frac{1}{n^2}$
$n = 2k$.
$\sum_{k=1}^{\infty} \frac{1}{(2k)^2} = \sum_{k=1}^{\infty} \frac{1}{4k^2} = \frac{1}{4} \underbrace{\sum_{k=1}^{\infty} \frac{1}{k^2}}_{\zeta(2)}$
$\zeta(2) = \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} + \frac{1}{4} \zeta(2)$

$$\frac{1}{n(n+1)(n+2)} = \frac{1/2}{n} + \frac{-1}{n+1} + \frac{1/2}{n+2}$$

$$= \frac{1/2}{n} - \frac{1/2}{n+1} - \frac{1/2}{n+1} + \frac{1/2}{n+2}$$

$$= \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+1} - \frac{1}{n+1} + \frac{1}{n+2}\right)$$

$$\frac{1}{2}\sum_{n=1}^{r}\left(\frac{1}{n} - \frac{1}{n+1}\right) - \frac{1}{2}\sum_{n=1}^{r}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$= \frac{1}{2}\left(1 - \frac{1}{r+1}\right) - \frac{1}{2}\left(\frac{1}{2} - \frac{1}{r+2}\right)$$

$$= \frac{1}{2} - \frac{1}{4} - \frac{1}{r+1} + \frac{1}{r+2}$$

$$= \frac{1}{4} - \frac{1}{r+1} + \frac{1}{r+2}$$

$$\lim_{r\to\infty} \sum_{n=1}^{r} a_n = \frac{1}{4} - 0 + 0$$

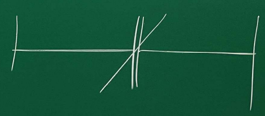
Aufgabe 3

Teil c

$$\frac{1}{f_n \cdot f_{n+2}} = \frac{f_{n+1}}{f_n \cdot f_{n+1} \cdot f_{n+2}} = \frac{f_{n+2} - f_n}{f_n \cdot f_{n+1} \cdot f_{n+2}} = \frac{\cancel{f_{n+2}}}{f_n \cdot f_{n+1} \cdot \cancel{f_{n+2}}} - \frac{\cancel{f_n}}{\cancel{f_n} \cdot f_{n+1} \cdot f_{n+2}} = \frac{1}{f_n \cdot f_{n+1}} - \frac{1}{f_{n+1} \cdot f_{n+2}}$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{f_n \cdot f_{n+2}} \Rightarrow \sum_{n=0}^{m} \frac{1}{f_n \cdot f_{n+1}} - \frac{1}{f_{n+1} \cdot f_{n+2}} = \sum_{n=0}^{m} \frac{1}{f_n \cdot f_{n+1}} - \sum_{n=0}^{m} \frac{1}{f_{n+1} \cdot f_{n+2}}$$

$$\Rightarrow \frac{1}{f_0 \cdot f_1} + \cancel{\frac{1}{f_m \cdot f_{m+1}}} - \cancel{\frac{1}{f_1 \cdot f_2}} - \frac{1}{f_{m+1} \cdot f_{m+2}}$$



$$\Rightarrow 1 - \frac{1}{f_{m+1} \cdot f_{m+2}} \xrightarrow{m \to \infty} 1$$

$f_m \geq 1$ für $m \in \mathbb{N}_0$

$f_{m+1} \geq f_m + 1$

$$\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n} = \sum_{n=1}^{\infty}\frac{(-1)^n}{2n} = -\frac{1}{2}+\frac{1}{4}-\frac{1}{6}+\frac{1}{8}-\frac{1}{10}+\cdots\cdots$$

$$= \sum_{n\in\{\text{ungerade}\}}^{\infty}\frac{(-1)^n}{2n} + \sum_{n\in\{\text{gerade}\}}^{\infty}\frac{1}{2n}$$

$$= \left(-1+\frac{1}{2}\right)+\frac{1}{4}+\left(-\frac{1}{3}+\frac{1}{6}\right)+\frac{1}{8}+\cdots$$

$$b_n = \begin{cases} a_{2m-1} = -\dfrac{1}{2m-1}, & \text{für } n = 3m-2 \\ a_{4m-2} = \dfrac{1}{4m-2}, & n = 3m-1 \\ a_{4m} = \dfrac{1}{4m}, & n = 3m \end{cases} \longrightarrow \text{Umordnung-satz \& Wohldefiniert}$$

$$b_n \longrightarrow \frac{1}{2}\cdot\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$$

$$\mathbb{N} \longrightarrow \mathbb{N}$$

$$m \longmapsto \begin{pmatrix} 3n-2 \\ 3n-1 \\ 3n \end{pmatrix}$$

$$\pi : \mathbb{N} \longrightarrow \mathbb{N}$$