$(n+1)! \quad (x-1)$

$$f^{(n+1)}(x) = (-1)^n \frac{n!}{x^{n+1}} \Rightarrow f^{(n+1)}(\xi) = (-1)^n \frac{n!}{\xi^{n+1}}$$

$|f^{(n+1)}(\xi)| = \frac{n!}{\xi^{n+1}}$ $\quad x \in [0.5, 1.5]$ gilt $|x-1| \le \frac{1}{2}$

$\xi \in [0.5, 1.5]$ gilt $\xi \ge \frac{1}{2}$

$$|f^{(n+1)}(\xi)| \le \frac{n!}{(\frac{1}{2})^{n+1}} = n!\,2^{n+1} \Rightarrow |R_n(x)| \le \frac{n!\,2^{n+1}}{(n+1)!} \left(\frac{1}{2}\right)^{n+1} = \frac{1}{n+1}$$

Aufgabe 1 $\quad f(x) = \ln(x)$

$f^{(k)}(x) = (-1)^{k-1} \frac{(k-1)!}{x^k}$ für $k \ge 1$

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x)}{k!}(x-1)^k$$

$$\frac{f^{(k)}(1)}{k!} = \frac{(-1)^{k-1}(k-1)!}{k!} = \frac{(-1)^{k-1}}{k} \Rightarrow T_n(x) = \sum_{k=1}^{n} (-1)^{k-1} \frac{(x-1)^k}{k}$$

(ii) $[0.5, 1.5]$

$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-1)^{n+1}$$

$$f^{(n+1)}(x) = (-1)^n \frac{n!}{x^{n+1}} \Rightarrow f^{(n+1)}(\xi) = (-1)^n \frac{n!}{\xi^{n+1}}$$

$$|f^{(n+1)}(\xi)| = \frac{n!}{\xi^{n+1}}$$

$x \in [0.5, 1.5]$ gilt $|x-1| \leq \frac{1}{2}$

$\xi \in [0.5, 1.5]$ gilt $\xi \geq \frac{1}{2}$

$$|f^{(n+1)}(\xi)| \leq \frac{n!}{\left(\frac{1}{2}\right)^{n+1}} = n!\,2^{n+1} \Rightarrow |R_n(x)| \leq \frac{n!\,2^{n+1}}{(n+1)!}\left(\frac{1}{2}\right)^{n+1} = \frac{1}{n+1}$$

Aufgabe 1 $\quad f(x) = \ln(x)$

$f^{(k)}(x) = (-1)^{k-1}(k-1)!$ für $k \geq 1$

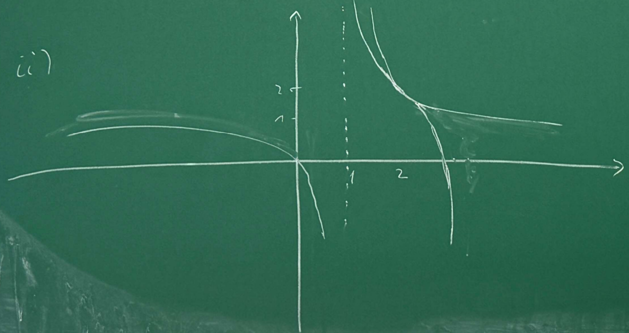
Aufgabe 2

i) $f(x) = \frac{x}{x-1}$ $\quad f'(x) = -\frac{1}{(x-1)^2}$ $\quad f''(x) = \frac{2}{(x-1)^3}$ $\quad f'''(x) = -6\,\frac{1}{(x-1)^4}$ $\quad f''''(x) = \frac{24}{(x-1)^5}$

$$T_{f,n,2}(x) = \sum_{i=0}^{3} \frac{f^{(i)}(2)}{i!}(x-2)^i = f(2) + f'(2)(x-2) + \frac{f''(2)}{2}\cdot(x-2)^2 + \frac{f'''(2)}{6}(x-2)^3$$

$$= -x^3 + 7x^2 - 17x + 16$$

ii)



iii) $R_{f,n}(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$

$$|R_{f,3}(x)| = \left|\frac{24}{(\xi-1)^5}\cdot\frac{1}{24}(x-2)^4\right| = \left|\frac{(x-2)^4}{(\xi-1)^5}\right|$$

$x \in [1.9, 2.1]$

$$\leq \frac{0.1^4}{(1.9-1)^5} \approx 0.00016935$$

iv) f'''' ist unbeschränkt.