$$t = \tan\left(\frac{x}{2}\right)$$

$$\frac{dt}{dx} = \frac{\frac{1}{2}\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) + \frac{1}{2}\sin\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} = \frac{1}{2} + \frac{1}{2}\tan^2\left(\frac{x}{2}\right) = \frac{1+\tan^2\left(\frac{x}{2}\right)}{2}$$

$$\sin(2x) = \frac{2\tan(x)}{1+\tan^2(x)}$$

$$\sin(x) = \frac{2t}{1+t^2} \qquad \cos(x) = \frac{1-t^2}{1+t^2}$$

$$\cos(2x) = \frac{1-\tan^2(x)}{1+\tan^2(x)}$$

$$\int \frac{1}{\cos(x)}\,dx = \int \frac{\cancel{1+t^2}}{1-t^2}\,\frac{2}{\cancel{1+t^2}}\,dt = 2\int \frac{1}{1-t^2}\,dt = 2\left(\int \frac{\frac{1}{2}}{1-t}\,dt + \int \frac{\frac{1}{2}}{1+t}\,dt\right)$$

$$= \ln|1+t| - \ln|1-t| + C = \ln\left|\frac{1+\tan\left(\frac{x}{2}\right)}{1-\tan\left(\frac{x}{2}\right)}\right| + C$$

$$\int_0^\infty e^{-x^2}$$

$$e^{-x^2} \leq e^{-x}$$

$$\int_1^\infty e^{-x} = e^{-1} < \infty$$

Für $x \in [0;1[$:

$$e^{-x^2} \leq 1$$

$$\int_0^1 \frac{1+\cos(x)}{x}\, dx$$

Da $\frac{\pi}{2} > 1$ und cos auf $[0, \frac{\pi}{2}]$ positiv ist, gilt für $x \in (0;1]$: $\frac{1+\cos(x)}{x} \geq \frac{1}{x} > 0$

$$\int \frac{1}{x} \longrightarrow +\infty \quad \text{divergent}$$

$$\int_0^\infty \cdots \quad , x \in [1, \infty[$$

$0 \leq x \leq 1$

$1 \leq x < \infty$

$\cos(x) \in [-1;1]$

$|\cos(x)| \leq 1$

$$t = \tan\left(\frac{x}{2}\right) \qquad \frac{dt}{dx} = \frac{\frac{1}{2}\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) + \frac{1}{2}\sin\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} = \frac{1}{2} + \frac{1}{2}\tan^2\left(\frac{x}{2}\right) = \frac{1+\tan^2\left(\frac{x}{2}\right)}{2}$$

$$\sin(2x) = \frac{2\tan(x)}{1+\tan^2(x)} \qquad \sin(x) = \frac{2t}{1+t^2} \qquad \cos(x) = \frac{1-t^2}{1+t^2}$$

$$\cos(2x) = \frac{1-\tan^2(x)}{1+\tan^2(x)}$$

$$\left(\int \frac{\frac{1}{2}}{\cdots} dt + \int \frac{\frac{1}{2}}{1+t} dt \right)$$

A3

$$\sum_{n=2}^{\infty} \frac{1}{n \log n^{\alpha}}$$

Integralkrit.

$$\sum_{n=0}^{\infty} a_n \text{ konv.} \iff \underbrace{\int_0^{\infty} f(x)\, dx}_{\text{konv}} \quad \text{mit } f(x) = a_x$$

a_n monoton fallend und $a_n \geq 0 \quad \forall n \in \mathbb{N}$

$$\int_2^{\infty} \frac{1}{x \log x^{\alpha}}\, dx \overset{\text{subst}}{=} \int_{\log 2}^{\infty} \frac{1}{u^{\alpha}}\, du = \left[\frac{u^{1-\alpha}}{1-\alpha}\right]_{\log 2}^{\infty}$$

Subst.

$u = \log x$

$du = \frac{1}{x}\, dx$

Fall $\alpha = 1$: Divergiert.

Fall $\alpha < 1$: $\lim_{u \to \infty} \overbrace{\frac{u^{1-\alpha}}{1-\alpha}}^{\to \infty} - \underbrace{\frac{\log 2^{1-\alpha}}{1-\alpha}}_{C \in \mathbb{R}} = \infty$

Fall $\alpha > 1$: $\lim_{u \to \infty} \underbrace{\frac{u^{\overset{<0}{1-\alpha}}}{1-\alpha}}_{=0} - \underbrace{\frac{\log 2}{1-\alpha}}_{C \in \mathbb{R}} = -C$

$$\sum_{n=2}^{\infty} \frac{1}{n\log(n)^{\alpha}}$$

$$\sum_{k=1}^{\infty} \frac{\cancel{2^k}}{\cancel{2^k}\log(2^k)^{\alpha}} = \sum_{k=1}^{\infty} \frac{1}{(k\log(2))^{\alpha}} = \frac{1}{\log(2)^{\alpha}} \sum_{k=1}^{\infty} \frac{1}{k^{\alpha}}.$$

$$\| \langle x, y \rangle \| \le \|x\|_p \; \|y\|_q$$

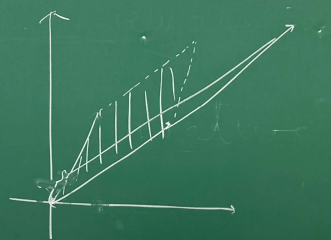
$$\|\cdot\|_\infty = \max\{\dots\}$$

$$\|\cdot\|_1 = |x_1| + |x_2| \dots$$

$$\|\cdot\|_n = \left(|x_1|^n + |x_2|^n + \dots + |x_n|^n\right)^{\frac{1}{n}}$$

$$\begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

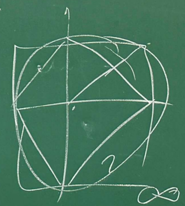
$$\|\cdot\| = \sqrt{\langle \cdot, \cdot \rangle}$$



$$\frac{1}{p} + \frac{1}{q} = 1$$

$\|x\|_p \ge \|x\|_q$

oder $p \le q$

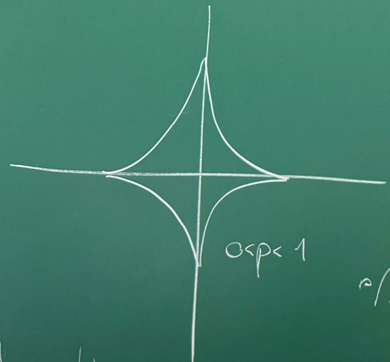


A3 $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \log n^{\alpha}}$

Integralkrit.

$\displaystyle\sum_{n=0}^{\infty} a_n$ konv. $\Leftrightarrow$ $\displaystyle\int_0^{\infty} f(x)\,dx$ konv. mit $f(x) = a_x$

$$\sum_{n=2}^{\infty} \frac{1}{n\log(n)^{\alpha}} \qquad \sum_{k=1}^{\infty} \frac{\cancel{2^k}}{\cancel{2^k}\log(2^k)^{\alpha}} = \sum_{k=1}^{\infty} \frac{1}{(k\log(2))^{\alpha}} = \frac{1}{\log(2)^{\alpha}} \sum_{k=1}^{\infty} \frac{1}{k^{\alpha}}.$$



$0<p<1$

$$\|\lambda \cdot v\|_p = |\lambda| \cdot \|v\|_p \qquad p \geq 1$$

$$\sqrt[p]{\lambda\,(\ldots)} \sim \sqrt[p]{\lambda\,(\ldots)} \qquad |\lambda|^p \cdot \|v\|_p \qquad p \in (0,1)$$

$$\|\lambda x\|_p = \left((|\lambda| x_1)^p + \cdots (\lambda x_k)^p\right)^{\frac{1}{p}} = \left(|\lambda|^p\right)\left((|x_1|^p + \cdots + |x_k|^p)\right)^{\frac{1}{p}} = |\lambda|\,\|x\|_p$$