Formelsammlung Mathematik: Unbestimmte Integrale trigonometrischer Funktionen

Zurück zu Formelsammlung Mathematik

Nachfolgende Liste enthält einige Integrale trigonometrischer Funktionen.

Die Konstante $c \in \mathbb{R}$ wird als ungleich 0 angenommen, und die Integrationskonstante wurde weggelassen.

Integrale trigonometrischer Funktionen, die sin enthalten

$$\int \sin cx \; dx = -\frac{1}{c} \cos cx$$

$$\int \sin^n cx \; dx = -\frac{\sin^{n-1} cx \cos cx}{nc} + \frac{n-1}{n} \int \sin^{n-2} cx \; dx \qquad (\, n > 0)$$

$$\int \sqrt{1 - \sin x} \, dx = 2 \frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \sqrt{1 - \sin x}$$

$$\int x \sin cx \; dx = \frac{\sin cx}{c^2} - \frac{x \cos cx}{c}$$

$$\int x^n \sin cx \; dx = -\frac{x^n}{c} \cos cx + \frac{n}{c} \int x^{n-1} \cos cx \; dx \qquad (\, n > 0)$$

$$\int_{\frac{-a}{2}}^{\frac{a}{2}} x^2 \sin^2 \frac{n\pi x}{a} \; dx = \frac{a^3 (n^2 \pi^2 - 6)}{24 n^2 \pi^2} \qquad (\, n = 2, 4, 6 ...)$$

$$\int \frac{\sin cx}{x} dx = \sum_{i=0}^{\infty} (-1)^i \frac{(cx)^{2i+1}}{(2i+1) \cdot (2i+1)!}$$

$$\int \frac{\sin cx}{x^n} dx = -\frac{\sin cx}{(n-1) x^{n-1}} + \frac{c}{n-1} \int \frac{\cos cx}{x^{n-1}} dx$$

$$\int \frac{dx}{\sin cx} = \frac{1}{c} \ln \left| \tan \frac{cx}{2} \right|$$

$$\int \frac{dx}{\sin^n cx} = \frac{\cos cx}{c(1-n) \sin^{n-1} cx} + \frac{n-2}{n-1} \int \frac{dx}{\sin^{n-2} cx} \qquad (\, n > 1)$$

$$\int \frac{dx}{1 \pm \sin cx} = \frac{1}{c} \tan \left(\frac{cx}{2} \mp \frac{\pi}{4} \right)$$

$$\int \frac{x\,dx}{1+\sin cx} = \frac{x}{c}\tan\left(\frac{cx}{2}-\frac{\pi}{4}\right) + \frac{2}{c^2}\ln\left|\cos\left(\frac{cx}{2}-\frac{\pi}{4}\right)\right|$$

$$\int \frac{x\,dx}{1-\sin cx} = \frac{x}{c}\cot\left(\frac{\pi}{4}-\frac{cx}{2}\right) + \frac{2}{c^2}\ln\left|\sin\left(\frac{\pi}{4}-\frac{cx}{2}\right)\right|$$

$$\int \frac{\sin cx\,dx}{1\pm\sin cx} = \pm x + \frac{1}{c}\tan\left(\frac{\pi}{4}\mp\frac{cx}{2}\right)$$

$$\int \sin c_1 x \sin c_2 x\,dx = \frac{\sin((c_1-c_2)x)}{2(c_1-c_2)} - \frac{\sin((c_1+c_2)x)}{2(c_1+c_2)} \qquad (\,|c_1| \neq |c_2|)$$

Integrale trigonometrischer Funktionen, die cos enthalten

$$\int \cos cx\,dx = \frac{1}{c}\sin cx$$

$$\int \cos^n cx\,dx = \frac{\cos^{n-1} cx \sin cx}{nc} + \frac{n-1}{n}\int \cos^{n-2} cx\,dx \qquad (\,n>0)$$

$$\int x\cos cx\,dx = \frac{\cos cx}{c^2} + \frac{x\sin cx}{c}$$

$$\int x^n \cos cx\,dx = \frac{x^n \sin cx}{c} - \frac{n}{c}\int x^{n-1}\sin cx\,dx$$

$$\int_{\frac{-a}{2}}^{\frac{a}{2}} x^2\cos^2\frac{n\pi x}{a}\,dx = \frac{a^3(n^2\pi^2-6)}{24n^2\pi^2} \qquad (\,n=1,3,5...)$$

$$\int \frac{\cos cx}{x}dx = \ln|cx| + \sum_{i=1}^{\infty}(-1)^i\frac{(cx)^{2i}}{2i\cdot(2i)!}$$

$$\int \frac{\cos cx}{x^n}dx = -\frac{\cos cx}{(n-1)x^{n-1}} - \frac{c}{n-1}\int \frac{\sin cx}{x^{n-1}}dx \qquad (\,n\neq 1)$$

$$\int \frac{dx}{\cos cx} = \frac{1}{c}\ln\left|\tan\left(\frac{cx}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \frac{dx}{\cos^n cx} = \frac{\sin cx}{c(n-1)cos^{n-1}cx} + \frac{n-2}{n-1}\int\frac{dx}{\cos^{n-2} cx} \qquad (\,n>1)$$

$$\int \frac{dx}{1-\cos cx} = -\frac{1}{c}\cot\frac{cx}{2}$$

$$\int \frac{x\,dx}{1+\cos cx} = \frac{x}{c}\tan\frac{cx}{2} + \frac{2}{c^2}\ln\left|\cos\frac{cx}{2}\right|$$

$$\int \frac{x\,dx}{1-\cos cx} = -\frac{x}{c}\cot\frac{cx}{2} + \frac{2}{c^2}\ln\left|\sin\frac{cx}{2}\right|$$

$$\int \frac{\cos cx\ dx}{1+\cos cx} = x - \frac{1}{c}\tan\frac{cx}{2}$$

$$\int \frac{\cos cx\ dx}{1-\cos cx} = -x - \frac{1}{c}\cot\frac{cx}{2}$$

$$\int \cos c_1 x \cos c_2 x\ dx = \frac{\sin((c_1-c_2)x)}{2(c_1-c_2)} + \frac{\sin((c_1+c_2)x)}{2(c_1+c_2)} \qquad (\,|c_1| \neq |c_2|)$$

Integrale trigonometrischer Funktionen, die tan enthalten

$$\int \tan cx\ dx = -\frac{1}{c}\ln|\cos cx| = \frac{1}{c}\ln|\sec cx|$$

$$\int \tan^n cx\ dx = \frac{1}{c(n-1)}\tan^{n-1} cx - \int \tan^{n-2} cx\ dx \qquad (\,n \neq 1)$$

$$\int \frac{dx}{\tan cx + 1} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx + \cos cx|$$

$$\int \frac{dx}{\tan cx - 1} = -\frac{x}{2} + \frac{1}{2c}\ln|\sin cx - \cos cx|$$

$$\int \frac{\tan cx\ dx}{\tan cx + 1} = \frac{x}{2} - \frac{1}{2c}\ln|\sin cx + \cos cx|$$

$$\int \frac{\tan cx\ dx}{\tan cx - 1} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx - \cos cx|$$

Integrale trigonometrischer Funktionen, die sec enthalten

$$\int \sec cx\, dx = \frac{1}{c}\ln|\sec cx + \tan cx|$$

$$\int \sec^n cx\, dx = \frac{\sec^{n-1} cx \sin cx}{c(n-1)} + \frac{n-2}{n-1}\int \sec^{n-2} cx\, dx \qquad (\,n \neq 1)$$

$$\int \frac{dx}{\sec x + 1} = x - \tan\frac{x}{2}$$

Integrale trigonometrischer Funktionen, die csc enthalten

$$\int \csc cx\, dx = -\frac{1}{c}\ln|\csc cx + \cot cx|$$

$$\int \csc^n cx\, dx = -\frac{\csc^{n-1} cx \cos cx}{c(n-1)} + \frac{n-2}{n-1}\int \csc^{n-2} cx\, dx \qquad (\,n \neq 1)$$

Integrale trigonometrischer Funktionen, die cot enthalten

$$\int \cot cx\, dx = \frac{1}{c} \ln|\sin cx|$$

$$\int \cot^n cx\, dx = -\frac{1}{c(n-1)} \cot^{n-1} cx - \int \cot^{n-2} cx\, dx \qquad (n \neq 1)$$

$$\int \frac{dx}{1+\cot cx} = \int \frac{\tan cx\, dx}{\tan cx + 1}$$

$$\int \frac{dx}{1-\cot cx} = \int \frac{\tan cx\, dx}{\tan cx - 1}$$

Integrale trigonometrischer Funktionen, die sowohl sin als auch cos enthalten

$$\int \frac{dx}{\cos cx \pm \sin cx} = \frac{1}{c\sqrt{2}} \ln\left|\tan\left(\frac{cx}{2} \pm \frac{\pi}{8}\right)\right|$$

$$\int \frac{dx}{(\cos cx \pm \sin cx)^2} = \frac{1}{2c} \tan\left(cx \mp \frac{\pi}{4}\right)$$

$$\int \frac{dx}{(\cos x + \sin x)^n} = \frac{1}{n-1}\left(\frac{\sin x - \cos x}{(\cos x + \sin x)^{n-1}} - 2(n-2)\int \frac{dx}{(\cos x + \sin x)^{n-2}}\right)$$

$$\int \frac{\cos cx\, dx}{\cos cx + \sin cx} = \frac{x}{2} + \frac{1}{2c} \ln|\sin cx + \cos cx|$$

$$\int \frac{\cos cx\, dx}{\cos cx - \sin cx} = \frac{x}{2} - \frac{1}{2c} \ln|\sin cx - \cos cx|$$

$$\int \frac{\sin cx\, dx}{\cos cx + \sin cx} = \frac{x}{2} - \frac{1}{2c} \ln|\sin cx + \cos cx|$$

$$\int \frac{\sin cx\, dx}{\cos cx - \sin cx} = -\frac{x}{2} - \frac{1}{2c} \ln|\sin cx - \cos cx|$$

$$\int \frac{\cos cx\, dx}{\sin cx(1+\cos cx)} = -\frac{1}{4c} \tan^2 \frac{cx}{2} + \frac{1}{2c} \ln\left|\tan \frac{cx}{2}\right|$$

$$\int \frac{\cos cx\, dx}{\sin cx(1+-\cos cx)} = -\frac{1}{4c} \cot^2 \frac{cx}{2} - \frac{1}{2c} \ln\left|\tan \frac{cx}{2}\right|$$

$$\int \frac{\sin cx\, dx}{\cos cx(1+\sin cx)} = \frac{1}{4c} \cot^2\left(\frac{cx}{2} + \frac{\pi}{4}\right) + \frac{1}{2c} \ln\left|\tan\left(\frac{cx}{2} + \frac{\pi}{4}\right)\right|$$

$$\int \frac{\sin cx\, dx}{\cos cx(1-\sin cx)} = \frac{1}{4c} \tan^2\left(\frac{cx}{2} + \frac{\pi}{4}\right) - \frac{1}{2c} \ln\left|\tan\left(\frac{cx}{2} + \frac{\pi}{4}\right)\right|$$

$$\int \sin cx \cos cx\, dx = \frac{1}{2c} \sin^2 cx$$

$$\int \sin c_1 x \cos c_2 x \, dx = -\frac{\cos(c_1+c_2)x}{2(c_1+c_2)} - \frac{\cos(c_1-c_2)x}{2(c_1-c_2)} \qquad (\,|c_1| \neq |c_2|)$$

$$\int \sin^n cx \cos cx \, dx = \frac{1}{c(n+1)} \sin^{n+1} cx \qquad (\,n \neq 1)$$

$$\int \sin cx \cos^n cx \, dx = -\frac{1}{c(n+1)} \cos^{n+1} cx \qquad (\,n \neq 1)$$

$$\int \sin^n cx \cos^m cx \, dx = -\frac{\sin^{n-1} cx \cos^{m+1} cx}{c(n+m)} + \frac{n-1}{n+m} \int \sin^{n-2} cx \cos^m cx \, dx \qquad (\,m, n > 0)$$

auch:

$$\int \sin^n cx \cos^m cx \, dx = \frac{\sin^{n+1} cx \cos^{m-1} cx}{c(n+m)} + \frac{m-1}{n+m} \int \sin^n cx \cos^{m-2} cx \, dx \qquad (\,m, n > 0)$$

$$\int \frac{dx}{\sin cx \cos cx} = \frac{1}{c} \ln|\tan cx|$$

$$\int \frac{dx}{\sin cx \cos^n cx} = \frac{1}{c(n-1)\cos^{n-1} cx} + \int \frac{dx}{\sin cx \cos^{n-2} cx} \qquad (\,n \neq 1)$$

$$\int \frac{dx}{\sin^n cx \cos cx} = -\frac{1}{c(n-1)\sin^{n-1} cx} + \int \frac{dx}{\sin^{n-2} cx \cos cx} \qquad (\,n \neq 1)$$

$$\int \frac{\sin cx \, dx}{\cos^n cx} = \frac{1}{c(n-1)\cos^{n-1} cx} \qquad (\,n \neq 1)$$

$$\int \frac{\sin^2 cx \, dx}{\cos cx} = -\frac{1}{c} \sin cx + \frac{1}{c} \ln\left|\tan\left(\frac{\pi}{4} + \frac{cx}{2}\right)\right|$$

$$\int \frac{\sin^2 cx \, dx}{\cos^n cx} = \frac{\sin cx}{c(n-1)\cos^{n-1} cx} - \frac{1}{n-1} \int \frac{dx}{\cos^{n-2} cx} \qquad (\,n \neq 1)$$

$$\int \frac{\sin^n cx \, dx}{\cos cx} = -\frac{\sin^{n-1} cx}{c(n-1)} + \int \frac{\sin^{n-2} cx \, dx}{\cos cx} \qquad (\,n \neq 1)$$

$$\int \frac{\sin^n cx \, dx}{\cos^m cx} = \frac{\sin^{n+1} cx}{c(m-1)\cos^{m-1} cx} - \frac{n-m+2}{m-1} \int \frac{\sin^n cx \, dx}{\cos^{m-2} cx} \qquad (\,m \neq 1)$$

auch: $$\int \frac{\sin^n cx \, dx}{\cos^m cx} = -\frac{\sin^{n-1} cx}{c(n-m)\cos^{m-1} cx} + \frac{n-1}{n-m} \int \frac{\sin^{n-2} cx \, dx}{\cos^m cx} \qquad (\,m \neq n)$$

auch: $$\int \frac{\sin^n cx \, dx}{\cos^m cx} = \frac{\sin^{n-1} cx}{c(m-1)\cos^{m-1} cx} - \frac{n-1}{n-1} \int \frac{\sin^{n-1} cx \, dx}{\cos^{m-2} cx} \qquad (\,m \neq 1)$$

$$\int \frac{\cos cx \, dx}{\sin^n cx} = -\frac{1}{c(n-1)\sin^{n-1} cx} \qquad (\,n \neq 1)$$

$$\int \frac{\cos^2 cx \, dx}{\sin cx} = \frac{1}{c}\left(\cos cx + \ln\left|\tan \frac{cx}{2}\right|\right)$$

$$\int \frac{\cos^2 cx \, dx}{\sin^n cx} = -\frac{1}{n-1} \left(\frac{\cos cx}{c \sin^{n-1} cx} + \int \frac{dx}{\sin^{n-2} cx} \right) \qquad (n \neq 1)$$

$$\int \frac{\cos^n cx \, dx}{\sin^m cx} = -\frac{\cos^{n+1} cx}{c(m-1) \sin^{m-1} cx} - \frac{n-m-2}{m-1} \int \frac{\cos^n cx \, dx}{\sin^{m-2} cx} \qquad (m \neq 1)$$

auch: $$\int \frac{\cos^n cx \, dx}{\sin^m cx} = \frac{\cos^{n-1} cx}{c(n-m) \sin^{m-1} cx} + \frac{n-1}{n-m} \int \frac{\cos^{n-2} cx \, dx}{\sin^m cx} \qquad (m \neq n)$$

auch: $$\int \frac{\cos^n cx \, dx}{\sin^m cx} = -\frac{\cos^{n-1} cx}{c(m-1) \sin^{m-1} cx} - \frac{n-1}{m-1} \int \frac{\cos^{n-2} cx \, dx}{\sin^{m-2} cx} \qquad (m \neq 1)$$

Integrale trigonometrischer Funktionen, die sowohl sin als auch tan enthalten

$$\int \sin cx \tan cx \, dx = \frac{1}{c} (\ln |\sec cx + \tan cx| - \sin cx)$$

$$\int \frac{\tan^n cx \, dx}{\sin^2 cx} = \frac{1}{c(n-1)} \tan^{n-1}(cx) \qquad (n \neq 1)$$

Integrale trigonometrischer Funktionen, die sowohl cos als auch tan enthalten

$$\int \frac{\tan^n cx \, dx}{\cos^2 cx} = \frac{1}{c(n+1)} \tan^{n+1} cx \qquad (n \neq -1)$$

Integrale trigonometrischer Funktionen, die sowohl sin als auch cot enthalten

$$\int \frac{\cot^n cx \, dx}{\sin^2 cx} = \frac{1}{c(n+1)} \cot^{n+1} cx \qquad (n \neq -1)$$

Integrale trigonometrischer Funktionen, die sowohl cos als auch cot enthalten

$$\int \frac{\cot^n cx \, dx}{\cos^2 cx} = \frac{1}{c(1-n)} \tan^{1-n} cx \qquad (n \neq 1)$$

Integrale trigonometrischer Funktionen, die sowohl tan als auch cot enthalten

$$\int \frac{\tan^m (cx)}{\cot^n (cx)} \, dx = \frac{1}{c(m+n-1)} \tan^{m+n-1}(cx) - \int \frac{\tan^{m-2} (cx)}{\cot^n (cx)} \, dx \qquad (m+n \neq 1)$$

Integrale trigonometrischer Funktionen mit symmetrischen Grenzen

$$\int_{-c}^{c} \sin x \; dx = 0$$

Siehe auch

- Englische Wikipedia (https://en.wikipedia.org/wiki/List_of_integrals_of_trigonometric_functions)